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This article compares, via synthetic data analysis, the performance of five different methods for scaling averaged dissimilarities data under conditions involving individual differences in "perception." All methods perform well when no "degradation" of the (simulated) ratings is entailed. When the data are transformed to zero-one values—a procedure sometimes followed in applied studies—all procedures perform poorly compared to the no-degradation case. Implications of these results for scaling applications involving group solutions are discussed.

Multidimensional Scaling and Individual Differences

To date, much of the applied research in multidimensional scaling has emphasized the scaling of group averages rather than scalings of separate individuals' dissimilarities judgments. This is certainly true in marketing research applications, although in some cases [7, 10], scalings have been made of subgroup data after the original subjects-by-dissimilarities matrix has been analyzed for individual differences by Tucker and Messick's points of view [16] model.

In some instances the data collection methods or scaling techniques used by marketing researchers preclude individual scalings. For example, Johnson [11] reports the use of a metric technique (multiple discriminant analysis) to find stimulus configurations in discriminant function space. The primary data consist of N individuals' numerical ratings of each of n stimuli with regard to each of m prespecified bipolar scales. The stimuli are assumed to represent groups in an n-way discriminant analysis of the data. A plot of the n stimulus centroids (taken across individuals) can be made in discriminant function space as a representation of the overall respondent group's perceptual configuration. Moreover, the normalized regression coefficients associated with average scale ratings (regressed on the discriminant axes) can be shown as vector directions in this same space as a guide to axis interpretation. However, individual differences are suppressed in this approach.[1] Additional problems associated with this method concern: (1) the choice of appropriate bipolar scales to begin with; (2) the possible inappropriateness of the linearity assumptions of the model; and (3) the implicit weights given to each scale in developing the squared distance measure (Mahalanobis' D^2) between pairs of stimulus centroids in the discriminant function space.

Stefflre [14] describes an alternative approach which can also lead to aggregation of data across respondents. In this case the primary data consist of a set of zero-one ratings by each of N respondents on n stimuli (products) with regard to m prespecified scales (e.g., use occasions for each product). Although full details of the computation procedure are not explicated, one *could* obtain a derived similarity score for each stimulus pair by aggregating, across individuals and profile components, the number of stimulus pair matchings (zero-zero or one-one). If this surmise is true (and Stefflre asserts that similarities responses are homogeneous over respondents anyway [15]), the multidimensional scaling analysis is again performed only at the overall group level.

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[1] This is not a necessity since the n-way discriminant analysis can be preceded by some type of cluster analysis which groups respondents into g groups, based on their relative similarity over the whole set of scale ratings. *Separate* discriminant analyses could then be made for each subgroup.

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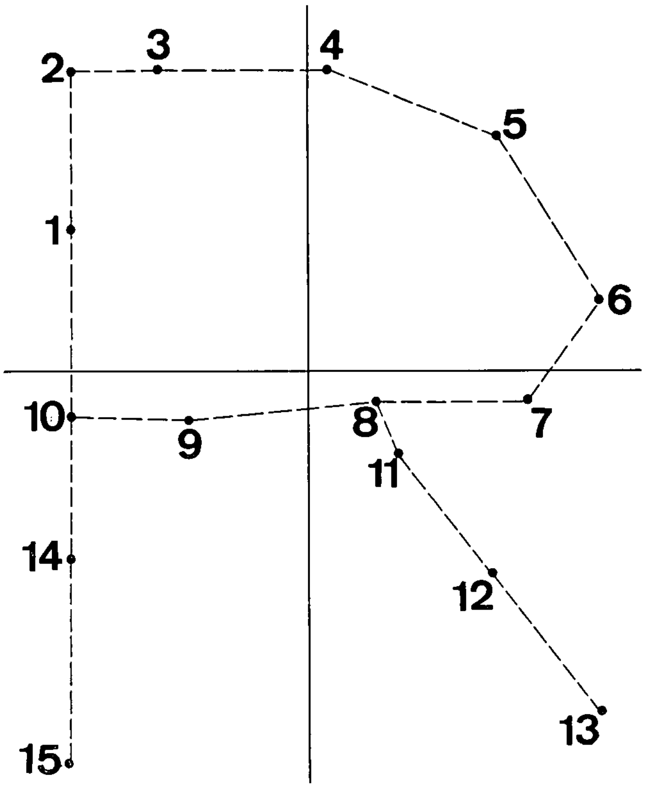
Figure 1
ORIGINAL GROUP STIMULUS CONFIGURATION



Howard and Sheth [10, pp. 212–20] describe a variety of models, all based upon Eckart and Young's decomposition model [6] and all utilizing stimulus ratings on prespecified attributes as primary input data. For our purposes, their approach can be viewed as representative of factor analytic methods. It should be mentioned, however, that Howard and Sheth discuss problems of individual differences in dissimilarities judgments and, indeed, utilize the Tucker and Messick approach [16] to determine homogeneous subgroups of respondents prior to (metric) scaling by means of factor analysis.

INDIVIDUAL DIFFERENCES MODELS

More recently, research interest by psychometricians has centered on the development of individual differences models in multidimensional scaling. Carroll and Chang [2], Bloxom [1], Horan [9], Kruskal [12], and McGee [13] have all proposed models for representing individual differences in dissimilarities judgments. Carroll and Chang have succinctly described the interrelationships among these models as well as the relationship of their model to Tucker and Messick's approach (which was the first of this class of models to appear in the literature).

Horan's, Bloxom's, and Carroll and Chang's models are quite similar conceptually,[2] although Horan's motivation is to show that metric scaling methods which develop the overall group's stimulus configuration by averaging individual estimates of interstimulus Euclidean distance (rather than using the root mean square of distance estimates) will lead to certain kinds of distortions of the group's normal attribute space [9]. If, however, the root mean square is used, the average individual's space will be related to the normal attribute space by, at worst, a linear transformation.

Both Carroll and Chang's and Bloxom's models result in a unique orientation of the group stimulus space (in Horan's terminology, the estimated normal attribute space). In addition, the models provide a set of individual subject weights whose square roots can be interpreted as coefficients which measure each respondent's differential stretching of the dimensions of the (common) group stimulus space in order to accommodate best his specific dissimilarity data. As currently formulated, each of the three models is metric, although Carroll and Chang show how their model can be made quasi-nonmetric in the sense of Young and Torgerson's first stage of their [17] two-stage approach to multidimensional scaling.

The motivation of the study reported here is similar to that of Horan. We wished to see, via synthetic data analyses, if a variety of procedures—including those suggested by researchers in marketing, viz., Johnson, Stefflre, and Howard and Sheth—could reproduce a known group stimulus configuration if the dimensions of this space were operated upon by individual respondents' (idiosyncratic) weights. The weights are assumed to reflect individual differences in salience regarding the dimensions of a common or group stimulus configuration.

THE STUDY DESIGN

The study design consisted of first constructing a group stimulus configuration, in two dimensions illustratively. For ease of visual interpretation, 15 points were arranged in two dimensions so as roughly to trace out the letter "R." Figure 1 shows this group stimulus configuration.

We next developed a set of individual subject weights or (squares of) axis-stretching factors so that each subject's transformed Euclidean distances could be represented as:

$$(1) \qquad d_{jk}^{(i)} = \left[\sum_{t=1}^{r} w_{it}(x_{jt} - x_{kt})^2 \right]^{1/2}$$

where (following, in part, the notation of Carroll and Chang):

w_{it} denotes the salience of Respondent $i (i = 1, 2,$

[2] It should be mentioned, however, that Horan's model, unlike those of Carroll and Chang and Bloxom, does not provide for a unique orientation of the stimulus configuration.

$\cdots, N)$ for dimension $t(t = 1, 2, \cdots, r)$ of the group stimulus configuration, and

x_{jt}, x_{kt} denote the projections of stimuli j and $k(j, k = 1, 2, \cdots, n; j \neq k)$ on dimension t of the group stimulus configuration.

We also assume here that the ith subject's dissimilarity measure $\delta_{jk}^{(i)}$ for the stimulus pair j and k is linearly related to $d_{jk}^{(i)}$, or notationally:

$$(2) \qquad \delta_{jk}^{(i)} = L(d_{jk}^{(i)})$$

where L represents a linear function with positive slope. Finally, in terms of the Carroll-Chang model we assume that stimulus j's coordinate value on axis t (in Subject i's transformed space) is represented by:

$$(3) \qquad y_{jt}^{(i)} = w_{it}^{1/2} x_{jt}$$

where:

$y_{jt}^{(i)}$ denotes the coordinate of stimulus j on dimension t of the configuration of Subiect i, and

$w_{it}^{1/2}$ denotes the square root of Subject i's salience for dimension t of the group stimulus space.

In this case each subject's space is assumed to represent a differential expansion (or contraction) along directions corresponding to the axes of the group stimulus space.

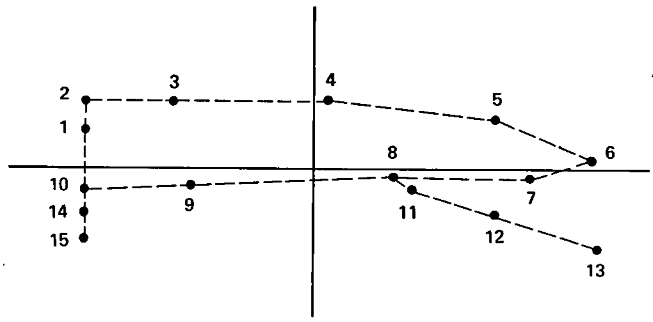
We next developed a set of 30 pairs of $w_{it}^{1/2}$ values—one for each axis of Figure 1—to represent 30 (hypothetical) subjects. The weights were chosen so as to vary systematically over the range $w_{i1}^{1/2} = 1$; $w_{i2}^{1/2} = 0$, to $w_{i1}^{1/2} = 0$; $w_{i2}^{1/2} = 1$. In all cases the sum of squares of the $w_{it}^{1/2}$ values was set to equal unity.[3] Figure 2 shows, illustratively, the differentially stretched space of Subject 5, using the square roots of his pair of salience weights, i.e., the $w_{it}^{1/2}$, as axis-stretching factors. These were 0.97662 and 0.21497, respectively, for dimensions one and two.

Thirty configurations, each representing a differential stretching or contraction of the group stimulus space, were constructed. Next, in order to simulate the notion of attribute ratings in the Johnson, Stefflre, and Howard-Sheth procedures, a set of 16 vectors was positioned in each (hypothetical) subject's individual space.[4] Vector directions (for each individual, independently) were chosen by: (1) selecting a random number between zero and $\pi/8$, representing an angle θ, expressed in radians; and (2) adding multiples of $\pi/8$ to the randomly determined starting angle so as to provide a reasonably full (and balanced) sweep of each subject's two-space.

Each subject's individual set of 16 ratings of each stimulus was then found by projecting the stimulus points onto his (idiosyncratic) set of vectors (and their reflections). For each subject, then, a 15 × 16 set of ratings (vector projections) was derived. (Note, however, that the rank of this matrix is two, given the manner in which the vectors were obtained.) Our final set of synthetic primary data thus consisted of a 30 × 15 × 16 three-way matrix (of subjects by stimuli by scales) whose cell entries were simulated numerical scale ratings.

Figure 2

SUBJECT 5'S DIFFERENTIALLY STRETCHED CONFIGURATION



Simulating the Scaling Methods

The three-way matrix of primary data was then scaled under a variety of procedures:

1. Johnson's procedure was simulated by running a 15-group discriminant analysis [5] of the ratings data. Each stimulus point was assumed to represent a group and the first two discriminant functions were found. Group centroids were plotted in this discriminant function space as a way of "recovering" the original configuration of Figure 1.
2. Euclidean distances (across the 16-component ratings profile) were next computed for each stimulus pair—first for each subject separately and then for the ratings averaged over all 30 subjects.
   a. The average subject's Euclidean distances were first scaled by a metric multidimensional scaling program, yielding a total-group stimulus configuration in two dimensions. In this case Euclidean distances were computed over the averaged profile ratings, *not* the root mean squared ratings.
   b. The average subject's Euclidean distances were then scaled by the TORSCA nonmetric scaling program, again yielding a total-group stimulus configuration. In this case only the rank order information in the input data was utilized.
   c. The 15 × 15 Euclidean distance matrices of each of the 30 subjects were submitted to Carroll and Chang's INDSCAL program to find a group stimulus space and a set of individual salience weights for each subject.
   d. The same data described in the last step were then scaled by Kruskal's approach to individual differ-

[3] The weights were chosen to result in equal axis weighting for the average subject. Thus, if procedures (root mean square of individual distances) suggested by Horan [9] were used, the original configuration of Figure 1 should be recovered perfectly.

[4] Given the manner in which vector directions were obtained, the last eight vectors are reflections (with perfect negative correlation) of the first eight. This redundancy was built into the model in order to simulate the tendency of many researchers to include scales with reversed polarity as a check on respondent consistency.

ences analysis, using the program option which constrains all subjects to share a common stimulus configuration but allows individual differences in the monotone function relating interpoint distances to input dissimilarities.

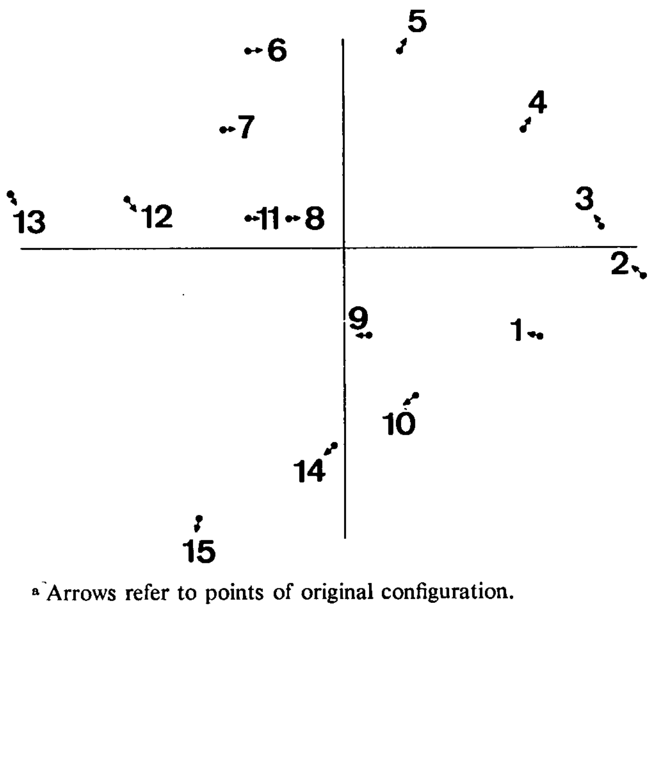
3. Stefflre's procedure was simulated by downgrading the ratings data of each subject to a set of zero-one ratings on each of the 16 vectors. Ratings above the mean of each scale were assigned a value of one and those less than or equal to the mean were assigned a value of zero. Next, a 15 $\times$ 15 similarities matrix was developed for each subject by simply counting the number of one-one and zero-zero matches for each pair of stimuli.
   a. These 30 similarities matrices were scaled by the same set of procedures described (in 2) above. (Aggregation was done by summing frequencies on a cell-by-cell basis.)
   b. Johnson's procedure was also used on these "degraded" data. In this case all individual ratings used in the discriminant analysis were again either zero or one.

As can be surmised from the above description, our principal objective was to examine the recovery of the known group stimulus configuration of Figure 1 under various methods that have been proposed by applied researchers in marketing. In addition, we wished to examine the individual difference approaches of Carroll and Chang and Kruskal under conditions designed to satisfy the Carroll-Chang model.

The problem of ascertaining the extent of (metric) recovery of the original configuration under the ten scaling analyses described above was handled by computing, as a goodness-of-fit measure, the product moment correlation between interpoint distances computed from the original configuration of Figure 1 and each of the recovered group stimulus configurations, respectively. While other measures could be used (e.g., an average cosine measure between the vectors of the original group stimulus configuration and the most congruent [4], orthogonally rotated recovered configuration, or a canonical correlation between the original configuration and each of the recovered configurations) the product moment correlation is the most appropriate, given the desire to retain information on relative interpoint distances. In this case we are restricting the type of transformation (linking recovered configuration to original configuration) to be a similarity transform.

Figure 3

RATINGS DATA[a]



[a] Arrows refer to points of original configuration.

RESULTS OF THE ANALYSIS

As indicated in the preceding section, numerical ratings were simulated by choosing 16 vectors in each of the 30 subjects' transformed spaces and projecting each subject's stimulus points onto each of his 16 rating vectors. The various scaling analyses described in the preceding section were then performed.

Ratings Data

We will first describe the results of the synthetic data analysis in terms of the metric models: (1) multiple discriminant analysis; (2) metric scaling of (average subject) interstimulus Euclidean distances, converted to scalar products; and (3) Carroll and Chang's INDSCAL program. From the first column of the table, all three approaches recover the group stimulus configuration essentially perfectly in terms of relative interpoint distance preservation. Illustratively, Figure 3 shows a plot of the original configuration (in a principal components orientation) and the configuration obtained by the multiple discriminant procedure, rotated to maximal congruence [4]. *All* of the techniques produced essentially equivalent results to those of Figure 3.

INTERPOINT DISTANCE CORRELATIONS BETWEEN ORIGINAL AND RECOVERED CONFIGURATIONS

Method	*Original ratings*	*Zero-one data*
Multiple discriminant analysis	0.997	0.738
Factor analysis of scalar products	1.000	0.309
INDSCAL analysis	0.999	0.756
TORSCA nonmetric scaling analysis	1.000	0.763
M-D-SCAL IV nonmetric analysis of individual differences	0.994	−0.025

The discriminant analysis, showing virtually perfect recovery of the group stimulus space, also yielded two discriminant functions which accounted perfectly for the total among-to-within-group dispersion.[5] While the metric (factor) analysis of scalar products (derived from Euclidean distances) accounted, in two dimensions, for slightly less than the total variance, the extra dimensions had no effect on solution recovery.[6] Since the underlying model was based on Carroll and Chang's INDSCAL formulation, it is not surprising that essentially perfect recovery of the group stimulus configuration was found here as well.

The nonmetric methods, requiring maintenance of only monotonicity between dissimilarities and distances, also produced essentially perfect reproduction of the original group stimulus space. The TORSCA average subject ratings (converted to dissimilarities by computing Euclidean distances of stimulus pairs in ratings space) and the M-D-SCAL IV approach (which allows idiosyncratic monotone transforms with common configuration over subjects) both revealed the linear character of the transformations used in the analysis. Not surprisingly, the stress value [17] associated with the TORSCA scaling was only 0.00005—for all practical purposes, zero.[7] The M-D-SCAL IV average stress value was considerably higher (0.064) but still yielded virtually perfect reproduction of the original group stimulus configuration.[8]

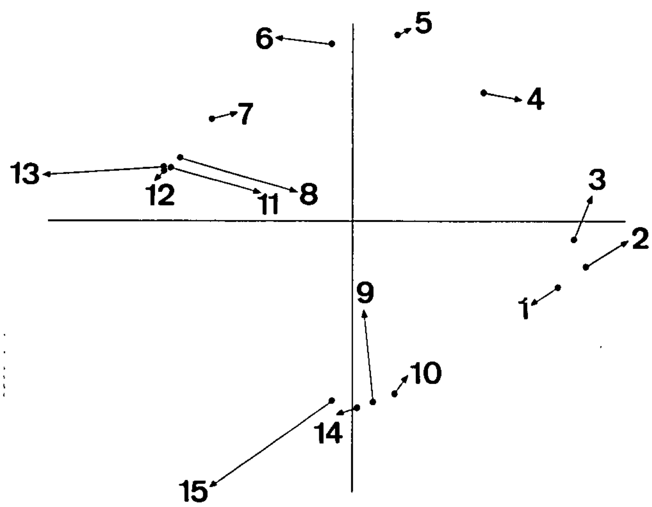
Zero-One Data

As will be recalled from the earlier discussion, in the second phase of the synthetic data analysis, the ratings data of each subject were downgraded to zero-one responses; ratings above the mean received a value of unity and those equal to or below the mean, zero.

In the multiple discriminant analysis the zero-one ratings were entered directly into the computer program. In the case of the remaining four scaling approaches similarity measures at the individual-subject level were computed by merely counting up the one-one and zero-zero matches for each stimulus pair across the 16 scales. In the case of the INDSCAL and M-D-SCAL IV models, the set of 30 such matrices was entered as similarities data. In the factor analytic and TORSCA analyses the similarities numbers were merely aggregated, cell by cell, across the 30 subjects. These cumulative similarity values then served as primary data input.

Figure 4

ZERO-ONE DATA



The second column of the table summarizes the recoveries associated with the downgraded data. Figure 4 shows a plot of the original configuration and that (illustratively) obtained from the multiple discriminant procedure, again rotated to maximal congruence with the original.

The multiple discriminant analysis of zero-one ratings yielded two large eigenvalues, accounting for 93% of the among-to-within-group dispersion and 11 more (quite small) roots, a reflection of the severe nonlinearities introduced by quantizing the numerical data. As would be expected, the correlation of interpoint distances between the original group stimulus configuration and that recovered by the discriminant procedure was only 0.738, in contrast to the essentially perfect recovery noted before.

The factor analysis of scalar products derived from the aggregated similarities data also reflected the nonlinearities introduced by the quantizing procedure. In this case three large eigenvalues, accounting for 94% of the variance, and five smaller roots were found. Again, we were unable to recover the "R" configuration; the interpoint distance correlation dropped markedly to 0.309. Apparently averaging distances (rather than squared distances) strongly affects recovery if the ratings are first subjected to a (severe, in this case) nonlinear transformation. This is so even though the correct dimensionality was utilized in the scaling of the similarity measures.

The quantizing procedure also affected the INDSCAL results, again a reflection of severe nonlinearities introduced in the data degradation. The interpoint distance

[5] Since the discriminant analysis implicitly utilized a squared distance measure (Mahalanobis' D^2) no extra dimensions of the sort described by Horan [9] are introduced, given data at the interval scaled level.

[6] Despite the fact that the procedure entailed an averaging of the square roots of the subject weights (rather than using the root mean square as suggested by Horan), the only effect of this incorrect procedure was to produce extra dimensions. The configuration found in the appropriate dimensionality (two dimensions) appeared to be unaffected. In this case, however, 99.8% of the total variance was accounted for by the first two dimensions.

[7] Since a nonmetric procedure was used here, use of an average of square roots of salience weights should not—and evidently did not—affect recovery of the original configuration.

[8] It should be pointed out that a nonarbitrary configuration (the original configuration itself) was used as a starting configuration for all M-D-SCAL IV analyses.

correlation dropped to 0.756 (about the same as noted in the discriminant analysis).

The nonmetric methods—TORSCA and M-D-SCAL IV—showed quite disparate results for the quantized data. The stress value under the TORSCA analysis, at 0.0045, was still low, and the highly nonlinear (essentially a step-function) transformation implied by the quantizing procedure was reproduced quite well. Still, the configuration that was produced by the program looked more like a circle than the letter "R." The relatively poor recovery was shown by an interpoint distance correlation of 0.763.

The M-D-SCAL IV results were considerably worse, however. First, the average stress found in this case was 0.5, even though a nonarbitrary starting configuration was used. The final configuration looked very little like the letter "R" and the nonlinear transform found above was not reproduced. Given these results, it is not surprising that the interpoint distance correlation was −0.025, showing no ability to recover the original configuration.

In summary, all methods performed essentially perfectly in the recovery of the group stimulus configuration in terms of the original ratings. Thus, despite the fact noted by Horan [9] that the averaging of distances (in this case computing distances across profiles based on an average of square roots of individuals' weights rather than root mean squares) can lead to nonlinear distortions of the group stimulus space, recoveries were practically perfect. As observed by Carroll and Chang [2], however, such distortions as described by Horan may be relatively minor *so long as the data are analyzed in the correct dimensionality.* (This is not to say, however, that the use of root mean squares is unnecessary; we merely wish to see how serious the effect would be if one departed from the correct averaging procedure.)

When the ratings were downgraded to zero-one values, *all* of the models displayed poorer recovery. The metric approaches, multiple discriminant analysis and INDSCAL, recovered the original configuration about as well as the TORSCA nonmetric procedure. The factor analytic (metric scaling) model and M-D-SCAL IV individual differences approach did considerably poorer. As noted above, Horan's suggestion regarding the use of appropriate averaging procedures turned out to be important in the case where the ratings were subjected to a nonlinear transformation prior to averaging.

DISCUSSION

Our comparison of five methods for obtaining group stimulus spaces under assumptions of individual perceptual differences (as portrayed by the Carroll-Chang, Bloxom, and Horan models) indicates that *all* procedures yield excellent recoveries before the data are "degraded" to zero-one ratings. Moreover, even the factor analysis of distances (converted to scalar products) as obtained from averaged ratings rather than root mean square ratings, reproduces the original configuration as well. Thus, using the appropriate dimensionality, distance averaging (rather than taking root mean squared distances) can provide a close approximation to results obtained by application of the (correct) method suggested by Horan [9], so long as the arguments of the distance function are not subjected to nonlinear transformation.

Quantizing the ratings data to zero-one values, however, results in a real loss in information, *despite* the fact that 16 ratings vectors were introduced in a manner so as to sweep the space of each individual subject's (transformed) configuration in a balanced way. At the average subject level the effect of this type of data degradation was to produce a configuration which looked more like a circle than the letter "R." The discriminant analysis, INDSCAL and TORSCA nonmetric analyses *all* produced circle-like configurations with consequent reduction in goodness of fit.[9] The recoveries found from the factor analysis and M-D-SCAL IV analysis were so poor as to be useless from a practical standpoint.

Thus we are led to conclude—at least tentatively—that the quantizing procedure results in a real loss of information at the two-space level which appears analogous to various types of information losses shown by Green and Rao [8] in the context of obverse factor analysis of correlation matrices.[10] The quantizing procedure used here exhibits the effect of projecting stimulus points onto a unit circle so that any two points falling on the same radius *cannot be distinguished after this transformation.* Not only is the dimensionality of the input data increased by this quantizing transformation, but lower dimensional information about interpoint distance is lost which cannot be recovered by *either* metric or nonmetric methods.

IMPLICATIONS

From a practical standpoint, this analysis has shown several findings which should be useful to application researchers:

1. Under conditions where no degradation of the ratings data takes place:
   a. The discriminant analysis procedure, utilized by Johnson [11], performs quite well and does not introduce extra dimensions associated with aver-

[9] A more appropriate recovery measure for the INDSCAL solution, given its unique orientation, would be the root mean square of pairwise projection correlations with the original configuration. Interpoint distance correlations were used here in order to be comparable to the procedure used in assessing recovery of the other scaling procedures.

[10] We might add that the results found here (for the average subject) were also found (in subsequent work) at the *individual* subject level; hence they do *not* appear to be a reflection of the averaging process as such.

aging distances rather than taking root mean squares.

b. The factor analytic proceured used by Howard and Sheth [10], does introduce extra dimensions but, if the correct dimensionality is used, the effect on configuration recovery is nil. Moreover, this procedure can be easily modified to use root mean squares, thus obviating any such difficulties.

2. Under conditions involving the quantizing of ratings data to zero-one values, all of the procedures used here—including nonmetric methods—lead to poor recovery of the original group stimulus configuration.

At this stage in the investigation of alternative scaling procedures, where individual differences in perception are quite possible, we support the type of model suggested by Carroll and Chang, Bloxom, and Horan. We take this view even though *all* of the above methods, at the ratings data level, lead to virtually perfect recoveries of the *group stimulus* space.

The Carroll-Chang model (which can be made quasi-nonmetric) not only develops the group stimulus space but, *in addition*, provides: (1) output information regarding individual differences in salience weights and (2) a *unique* orientation of the group stimulus space. This additional information strikes us as highly relevant for investigating the bases of individual differences in dissimilarities judgments, their relationship to market segmentation and associated policy questions in marketing management.

REFERENCES

1. Bloxom, Bruce. *Individual Differences in Multidimensional Scaling*, Research Bulletin 68-45. Princeton: Educational Testing Service, 1968.
2. Carroll, J. Douglas and Jih J. Chang. "Analysis of Individual Differences in Multidimensional Scaling via an *N*-way Generalization of 'Eckart-Young' Decomposition," paper presented at the Spring meeting of the Psychometric Society, Princeton, April 1969.
3. ———. "Individual Differences and Multidimensional Scaling." Murray Hill, N.J.: Bell Telephone Laboratories, 1969 (mimeographed).
4. Cliff, Norman. "Orthogonal Rotation to Congruence," *Psychometrika*, 31 (March 1966), 33–42.
5. Dixon, W. J., ed. *Biomedical Computer Programs*. Berkeley: Health Sciences Facility, University of California, 1965.
6. Eckart, Charles and Gale Young. "The Approximation of One Matrix by Another of Lower Rank," *Psychometrika*, 1 (September 1936), 211–8.
7. Green, Paul E., Arun Maheshwari, and Vithala R. Rao. "Dimensional Interpretation and Configuration Invariance in Multidimensional Scaling: An Empirical Study," *Multivariate Behavioral Research*, 6 (April 1969), 159–80.
8. Green, Paul E. and Vithala R. Rao. "A Note on Proximity Measures and Cluster Analysis," *Journal of Marketing Research*, 6 (August 1969), 359–64.
9. Horan, Charles B. "Multidimensional Scaling: Combining Observations When Individuals Have Different Perceptual Structures," *Psychometrika*, 34 (June 1969), 139–65.
10. Howard, John A. and Jagdish N. Sheth. *The Theory of Buyer Behavior*. New York: John Wiley & Sons, 1969.
11. Johnson, Richard M. "Market Segmentation—A Comparison of Techniques," paper presented at the 16th International Meeting of the Institute of Management Sciences, New York, March 1969.
12. Kruskal, Joseph B. "How to Use M-D-SCAL, A Program to do Multidimensional Scaling and Multidimensional Unfolding" (Versions 4 and 4M of M-D-SCAL, all in FORTRAN IV). Murray Hill, N.J.: Bell Telephone Laboratories, 1968 (mimeographed).
13. McGee, Victor E. "CEMD/DEMD Nonmetric Individual Differences Model for (Elastic) Multidimensional Data Reduction—to Handle N Sets of Multivariate Data," *Journal of Marketing Research*, 5 (August 1968), 322.
14. Stefflre, Volney J. *Some Materials Describing Market Structure Studies*. Laguna Beach, Calif.: Stefflre Associates, Inc., 1968.
15. Bass, F. M., C. W. King, and E. A. Pessemier, eds. *Application of the Sciences in Marketing Management*. New York: John Wiley & Sons, 1968, 255–6.
16. Tucker, Ledyard R. and Samuel Messick. "An Individual Differences Model for Multidimensional Scaling," *Psychometrika*, 28 (December 1963) 333–67.
17. Young, Forrest W. and Warren S. Torgerson. "TORSCA, A FORTRAN IV Program for Shepard-Kruskal Multidimensional Scaling Analysis," *Behavioral Science*, 12 (November 1967) 498.